NATIONAL GUARD
MOBILE LEARNING CENTER

WATER ENERGY

Factoring

Grade 12 | Mathematics
Concept: Quadratic Equations | Level: I/II

Connections to the Energy Lab:

- The Water Experience focuses on harnessing energy through wave production in open water

General Goal(s) of the Lesson:

Students should be able to…

- Explain why factoring can be used to solve quadratic equations in the form $ax^2 + bx + c = 0$
- Factor quadratic polynomials or state that a polynomial cannot be factored over a number set (real numbers, rational numbers, imaginary numbers)

Lesson Objectives:

- Show methods of factoring (greatest common factor, difference of squares, sum/product, factoring by grouping) and ways to identify when to use each

(Based on Massachusetts State Standard 12.P. 7 for Mathematics)

Assessment Based on Objectives:

- Given a quadratic expression, factor into binomials or another pair of factors
- Evaluate whether a quadratic expression can be factored using the discriminant
- Create a quadratic polynomial that can be factored by multiplying and then trade with a partner and have him/her factor

Learning Adaptations:	Learning Extensions:
- Have students identify which method to use (but do not factor) in order to isolate the skill	- Have students factor non-numerical quadratics ($a^2 – b^2$) - Recognize other simple forms of factorable polynomials such as differences of cubes

Resources:

Let's Review: Math A by Lawrence S. Leff (ISBN: 0764122967)
Head First Algebra: A Learner's Guide to Algebra I by Dan Pilone and Tracey Pilone (ISBN: 0596514867)

NATIONAL GUARD
MOBILE LEARNING CENTER

WATER ENERGY
Graphing Parabolas

Grade 12 | Mathematics
Concept: Quadratic Equations | Level: III

Connections to the Energy Lab:
- The Water Experience focuses on harnessing energy through wave production in open water

General Goal(s) of the Lesson:
Students should be able to…
- Plot points from a quadratic function given a domain
- Locate the x-intercepts of a function from the graph or by solving f(x) = 0 algebraically
- Produce a sketch of the graph of a quadratic function, including information about intercepts, axis of symmetry, and concavity/orientation

Lesson Objectives:
- Have students generate function values for a quadratic function of the form f(x) = ax2 + bx + c for specific x-values
- Use graphing technology to demonstrate the shapes of the graph for various functions
- Help students understand that an intercept can be located by the value at which the graph crosses an axis, or by the value of one variable when the other is set equal to zero
- Students should recognize the symmetry of the graph of a quadratic function, and use the axis of symmetry to provide additional points to be graphed (assuming they are part of the given domain)

(Based on Massachusetts State Standard 12.P. 7 for Mathematics)

Assessment Based on Objectives:
- Students can be given a number of graphs and match equations to the graphs
- Given a function, domain, and graph paper, students should scale axes and produce a graph containing at least five accurately plotted points
- Based on plotted points or a complete graph of a quadratic function, students should be challenged to write an equation for the function

Learning Adaptations:	Learning Extensions:
- Graphing technology can be used to check graphs or generate tables of ordered pairs for students having trouble with arithmetic	- Other functions (polynomial, logarithmic, exponential) can be graphed by generating a table of values and plotting them - Relate quadratic functions to other even functions with the property that f(-x) = f(x)

Resources:
Let's Review: Math B by Lawrence S. Leff (ISBN: 0764116568)
Pre-Calculus: Building Concepts and Connections by Revathi Narasimhan (ISBN: 0618413014)

NATIONAL GUARD
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WATER ENERGY

Identifying Sine, Cosine and Tangent Lines

Grade 12 | Mathematics
Concept: Basic Trigonometry | Level: I/II

Connections to the Energy Lab:

- The Water Experience focuses on harnessing energy through wave production in open water

General Goal(s) of the Lesson:

Students should be able to…

- Determine whether or not a graph is a periodic trigonometric function (sine, cosine or tangent)
- Identify quantities such as amplitude, frequency, and period for a wave
- Explain how sine, cosine or tangent functions can be used to model periodic phenomena

Lesson Objectives:

- Introduce concept of periodic motion, relating it to objects floating in the ocean
- Relate frequency of oscillation to power generation. Frequency measures how often an object bobs up and down
- Identify amplitude, period, and frequency of sine and cosine functions. Explain similarities and differences of the tangent function

(Based on Massachusetts State Standard 12.G.1 & 12.G.2 for Mathematics)

Assessment Based on Objectives:

- Students can classify a set of graphs being periodic but not trigonometric, neither periodic or trigonometric, or pure trigonometric functions
- Using real world data (ex. output voltage of a solar panel during the day) students can create a numerical model using a sine and cosine function

Learning Adaptations:	Learning Extensions:
- Show an object bouncing on a spring and identify frequency, period, and amplitude for its motion before introducing the abstract concept of the periodic function	- Apply concept to electrical signals, oscillation, vibration, and daily fluctuation in absorbed light on solar panels - Advanced students can be introduced to the concept of phase shift that can turn a sine function into a cosine function

Resources:

Sin/Cos/Tan Online Applet Functions 3
http://www.univie.ac.at/future.media/moe/galerie/fun2/fun2.html#sincostan
REA's Ready, Set, Go! Trigonometry Workbook (REA) (Test Preps) (ISBN: 0738604550)
http://www.amazon.com/REAs-Ready-Trigonometry-Workbook Preps/dp/0738604550/ref=sr_1_2?ie=UTF8&s=books&qid=1279809047&sr=1-2
Interactive Graph of Sin/Cos/Tan Online Applet
http://mathforum.org/mathtools/cell/tr,13.2.2,ALL,ALL/

NATIONAL GUARD
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WATER ENERGY

Graphing Sine, Cosine and Tangent Lines

Grade 12 | Mathematics
Concept: Basic Trigonometry | Level: III

Connections to the Energy Lab:

- The Water Experience focuses on harnessing energy through wave production in open water

General Goal(s) of the Lesson:

Students should be able to…

- Describe how the coefficients and constraints in A sin (bx + c) + D change the graph of the sine function in standard form. This should also include equivalent transformations of the cosine function in standard form.
- Sketch a graph of a sine, cosine, or tangent function given the equation in standard form
- Describe the maximum and minimum values, frequency, and period of a function given the equation

Lesson Objectives:

- Review concepts that make the graphs of trigonometric functions unique (i.e. periodicity and amplitude)
- Show that amplitude comes from the height above/below the middle value (which is the constant D)
- Period comes from 2pi/b. This can be used to determine at which value of x the function repeats
- Demonstrate that this information is sufficient for sketching the graph of a function

(Based on Massachusetts State Standard 12.G.1 & 12.G.2 for Mathematics)

Assessment Based on Objectives:

- Students can sketch a graph of a trigonometric function with given criteria (amplitude, frequency, period)
- Given a set of graphs, students can match the equation to the function
- Students can sketch a single period of a sine or cosine function given the equation

Learning Adaptations:	Learning Extensions:
- Drawing a box to help students with a single period of the wave: width is one period, height is the min/max of the amplitude	- Tangent function graphs can also be graphed by connecting the locations of asymptotes and intercepts to the unit circle

Resources:

Lesson Plans Graphs from a Unit Circle http://illuminations.nctm.org/LessonDetail.aspx?id=L785
Interactive Tutorial Applet http://www.analyzemath.com/unitcircle/unitcircle.html
Sine, Cosine & Tangent Functions Dynamic Online Worksheet
http://skyview.billings.k12.mt.us/departments/math/riehl/geogebra/sincostan2/sincostan2.html

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SOLAR ENERGY

Relating Sides of Triangle to Trigonometric Ratios

Grade 12 | Mathematics
Concept: Inverse Operations | Level: I/II

Connections to the Energy Lab:
- The Fire Experience focuses on capturing solar energy via solar panels, solar arrays and solar farms

General Goal(s) of the Lesson:
Students should be able to…
- Calculate the lengths of sides of a right triangle using trigonometric ratios of angles in that triangle
- Solve word problems that require use of trigonometric ratios to find lengths of sides in right triangles

Lesson Objectives:
- Show how trigonometric ratios can be calculated using the Pythagorean Theorem and the ratios from the acronym S(OH)C(AH)T(OA)
- Students should discover that in similar triangles, the angles are the same, and the trigonometric ratios corresponding to each acute angle are equal
- The fact that all right triangles with congruent angles are similar is what makes trigonometric ratios the same for similar triangles of different sizes
- Demonstrate that the calculator's ability to calculate trigonometric ratios for angles allows unknown sides of right triangles to be determined
- Emphasize the importance of drawing correct diagrams in word problems

(Based on Massachusetts State Standard 12.P.5 for Mathematics)

Assessment Based on Objectives:
- Students should be able to calculate an unknown side of a right triangle given an angle and a known side length
- Have students draw a triangle for a given word problem and show how trigonometry can be used to determine an unknown side length
- Students can calculate the height of the school, a tree, a wind turbine, or any other object using a protractor and hanging object (to measure angle of elevation), a tape measure, and calculations. Students doing this activity are writing their own word problems!

Learning Adaptations:
- Students write out the adjacent, opposite and hypotenuse lengths before calculating any ratios to practice identifying them
- Introduce concept of angle of elevation/depression for word problems using similar words (elevate = "raise up", depressed = "be down", etc.)

Learning Extensions:
- The trigonometric ratios originate from the unit circle
- Students should be trained to recognize important triangles (30-60-90, 45-45-90) and their relation to sines and cosines on the unit circle
- Pre-Calculus students can use the laws of sines and cosines to solve problems of this type

Resources: http://www.wonderhowto.com/how-to-find-area-triangle-when-given-2-sides-angle-302122/

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SOLAR ENERGY

Converting Sides of a Triangle into a Trigonometric Ratio

Grade 12 | Mathematics
Concept: Inverse Operations | Level: III

Connections to the Energy Lab:

- The Fire Experience focuses on capturing solar energy via solar panels, solar arrays and solar farms

General Goal(s) of the Lesson:

Students should be able to…

- Use trigonometric ratios and side lengths to calculate measures of interior angles of right triangles
- Solve word problems that require use of trigonometric ratios to find measures of angles in right triangles
- Recognize that sin(x) and arcsin(x)/$\sin^{-1}$(x) are inverse functions

Lesson Objectives:

- Remind students that a single side length can be unknown and found given the angle and a second side length
- In the same way, an unknown angle can be determined using two given side lengths. Students should see that the angle given by the calculator is always between 0 and 90 degrees
- Emphasize the importance of drawing correct diagrams for word problems
- Show several examples of triangles with unknown angles or sides to make students decide when to use sin(x), when to use $\sin^{-1}$(x), and when to use the Pythagorean Theorem
- Students should always include the argument of the trig functions (sin(x) = 3/5, not sin = 3/5) not just for mathematical accuracy, but to avoid confusion when switching between unknown angles and unknown side length problems

(Based on Massachusetts State Standard 12.P.5 for Mathematics)

Assessment Based on Objectives:

- Calculate the angle of elevation for an object hung up in the classroom and use a laser pointer with a protractor to verify the calculation
- Students should calculate the angle of elevation for projectiles that reach a known height (i.e. fireworks, model rockets) and then measure the angle to see how close the prediction was
- Solve problems such as finding the cosine for angle x in a right triangle with sin(x) = 3/5. Using the calculator in problems like this is useful, but understanding the use of function inverses allows an exact answer to be determined.
- Calculating orientation of vectors given the components using $\tan^{-1}$(x).

Learning Adaptations:	Learning Extensions:
- Students should become comfortable writing the trig ratios with variables or numerical values so that finding angles seems like a similar problem to finding side lengths	- The inverse trigonometric functions use a restricted domain to ensure that the inverse is a function

Resources: Worksheets http://www.funmaths.com/worksheets/math_trigonometry_05.htm

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EARTH, WIND, SOLAR & WATER ENERGY

Graphing a Set of Data

Grade 12 | Mathematics

Concept: Frequency Distribution of Data | Level: I/II

Connections to the Energy Lab:

- The Earth Experience focuses on balancing worldwide fossil fuel demands vs. energy output
- The Wind Experience focuses on the different variables that affect wind turbine performance
- The Fire Experience focuses on capturing solar energy via solar panels, solar arrays and solar farms
- The Water Experience focuses on harnessing energy through wave production in open water

General Goal(s) of the Lesson:

Students should be able to…

- Construct a frequency table and a histogram for a given set of data
- Interpret a histogram and what it indicates about the set of data

Lesson Objectives:

- Students should understand that statistics is about representing large amounts of data using a few carefully selected numbers
- Tables of data are difficult to interpret, especially with a large set of data, so frequency graphs are often used to show the entire data set at once

(Based on Massachusetts State Standard 12.D.2 & 12.D. 5 for Mathematics)

Assessment Based on Objectives:

- Given a set of data, students should be able to produce a frequency table and histogram and then describe the minimum and maximum for the data set, the mode, and the mean based on the graph
- Students can conduct a survey and construct a histogram/frequency table of the results. There should be a minimum of 15 elements in the data set to get a meaningful data set. A good discussion can result on why larger data sets are better for statistical analysis.

Learning Adaptations:

- Students should use tally marks to keep track of elements of a data set. This can be an additional column in the frequency table
- To show why the graph is better than a raw table of data, give a graph of a set to one group and the table to another, and then ask them to both identify the max, min and mode. The graph group inevitably wins and will prove the logic behind its use.

Learning Extensions:

- Students could obtain real wind turbine (wind speed) data for a site nearby and construct a graph for the location
- A discussion of mean, median and mode can precede or follow a discussion on frequency distributions
- Discuss the concept of uniform or normal distribution

Resources:

Power Of: Appleworks/Data Base Spread Sheet Word Processing/With Disk ISBN: 9996458903

Material Environmental Data Sheets ISBN: 0471286583

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EARTH, WIND, SOLAR & WATER ENERGY

Identifying Spread, Skewness, Symmetry and Modes

Grade 12 | Mathematics

Concept: Frequency Distribution of Data | Level: III

Connections to the Energy Lab:

- The Earth Experience focuses on balancing worldwide fossil fuel demands vs. energy output
- The Wind Experience focuses on the different variables that affect wind turbine performance
- The Fire Experience focuses on capturing solar energy via solar panels, solar arrays and solar farms
- The Water Experience focuses on harnessing energy through wave production in open water

General Goal(s) of the Lesson:

Students should be able to…

- Qualitatively describe the spread and skewness/symmetry of a frequency distribution
- Identify the mode of a data set from the graph and use technology to calculate standard deviation

Lesson Objectives:

- Given graphs that have different levels of skewness or symmetry, students should see that there are more elements of the data set occurring to one side or another of the mean. When in doubt with a graph, students can always go back to what the graph shows about the original data set.
- It can also be useful to see that skewed data sets can have big differences between their means, medians and modes. Symmetric data sets have all three numbers close together.
- Show two graphs with the same mean but different amounts of spread. Students will likely be able to easily describe the physical differences between the graphs, but describing the significance of this for the data can be more difficult. If needed, go back to two data sets of different amounts of spread, and have students describe the difference between the two.
- Understand how standard deviation is a measure of spread and variation in data, and be familiar with calculating it using a calculator or computer software

(Based on Massachusetts State Standard 12.D.2 & 12.D. 5 for Mathematics)

Assessment Based on Objectives:

- Given two frequency distributions with different amount of spread, have students identify which has the larger standard deviation
- Give a real world example with a skewed data set and ask students how it would compare to a symmetric distribution. If they understand the concept, they should use language like "the data set had more data above the mode than below…"
- Ask students to design a survey or an experiment that involves taking large amounts of data. Students cannot only create frequency tables and histograms for the data, but also describe the variation, spread, and amount of skewness in the results.

Learning Adaptations:	Learning Extensions:
- The term spread on a frequency distribution can be compared to the physical meaning of spread. A distribution with low spread will NOT be spread out. - Students should be familiar with symmetry. Always go back to the graph to evaluate whether it is symmetric or not.	- Include a discussion of outliers, both by definition and by concept - Describe the significance of a skewed wind speed data set, in which it is better to be skewed towards high speed for better wind power generation.

Resources: Symmetry: A Journey into the Patterns of Nature by Marc Du Sautoy